$$\text{Then } 17^3\times73^5=\begin{vmatrix}493 & 340\\ -340 & 493\end{vmatrix}\times\begin{vmatrix}5329 & 0\\ 0 & 5329\end{vmatrix}=(493^2+340^2)(5329)^2$$

$=(73)^2(493)^2+(73)^2(340)^2$, which is one set of answers.

Also solved by *R. J. Adcock, and M. A. Gruber.*

19. Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C

Find three positive integer numbers whose sum is a cube, and, also, the sum of any two diminished by the third a cube.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x, y, and z=the three positive integers.

$$\text{Then }\begin{aligned}x+y+z&=a^3\\ x+y-z&=b^3\\ x+z-y&=c^3\\ y+z-x&=d^3\end{aligned}$$

$$\text{Whence } x+y+z=b^3+c^3+d^3=a^3;$$

$$x=\frac{b^3+c^3}{2};\ y=\frac{b^3+d^3}{2};\ z=\frac{c^3+d^3}{2}.$$

This is a problem in which the sum of three cubes=a cube. Take $3^3+4^3+5^3=6^3$. But as the numbers are to be integers, we multiply by 2, and obtain $6^3+8^3+10^3=12^3$. $\therefore$ $x=\dfrac{6^3+8^3}{2}=364$; $y=\frac{1}{2}(6^3+10^3)=608$; and $z=\frac{1}{2}(8^3+10^3)=756$. The number of answers is infinite.

II. Solution by R. J. ADCOCK, Larchland, Warren County, Illinois.

Let x, y, z, be the three numbers; then $x+y+z=u^3$, $x+y-z=v^3$, $x+z-y=r^3$, $z+y-x=s^3$, by conditions.

Wherefore $x+y+l=u^3=v^3+r^3+s^3$, $x=\dfrac{v^3+r^3}{2}$, $y=\frac{1}{2}(v^3+s^3)$, $z=\frac{1}{2}(r^3+s^3)$. The most general equation yet obtained by me for the sum of three cubes=a cube, is found from,

$[(ax^3+dy^3)x]^3+[(bx^3+hy^3)y]^3+[(cx^3-hy^3)y]^3=[(ax^3+gy^3)x]^3$, by expanding, equating coefficients of similar terms with respect to x and y, eliminating d, g, and h, giving the identical equation,

$$[9a^3bx^3y+(b^2-bc+c^2)^2y^4]^3+[9a^3cx^3y-(b^2-bc+c^2)^2y^4]$$
$$+[9a^4x^4-3(b^2-bc+c^2)caxy^3]^3=[9a^4x^3+3ab(b^2-bc+c^2)xy^3]^3.$$

By numbers for the letters in the above, some of the resulting

equations are $3^3+4^3+5^3=6^3$, $1^3+6^3+8^3=9^3$, $3^3+10^3+18^3=19^3$,

$$7^3+14^3+17^3=20^3,\ 4^3+17^3+22^3=25^3,\ 11^3+15^3+27^3=29^3.$$

Then the three positive integer numbers are $x=\dfrac{11^3+15^3}{2}$, $y=\dfrac{11^3+27^3}{2}$,

$z=\dfrac{15^3+27^3}{2}$. Also x, y, z may be found from any equation, including the algebraicsum, for the sum of three cubes=a cube, by first multiplying each cube by 2^3.

Also solved by *O. W. Anthony, H. W. Draughon, C. D. Schmitt, and G. B. M. Zerr.*

PROBLEMS.

27. **Proposed by J. W. NICHOLSON, LL. D., President and Professor of Mathematics, Louisiana State University and A. and M. College, Baton Rouge, Louisiana.**

Required a formula for finding five integers the sum of whose cubes is a cube.

28. **Proposed by DAVID E. SMITH, Ph. D., Professor of Mathematics, Michigan State Normal School, Ypsilanti, Michigan.**

Decompose the product 97.673.257 into the sum of two fourth powers.

AVERAGE AND PROBABILITY.

Conducted by B.F.FINKEL, Kidder, Missouri. All Contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

13. **Proposed by I. L. BEVERAGE, Monterey, Virginia.**

Find the mean values of the roots of the quadratic $x^2-ax+b=0$, the roots being known to be real, but b being unknown and positive.

Solution by P. S. BERG, Apple Creek, Ohio, and JOHN DOLMAN, Jr., Counsellor-at-law, Philadelphia, Penn, and J. M COLAW, A. M., Principal of High School, Monterey, Virginia.

Solving the given equation, $x=\frac{1}{2}a\pm\sqrt{(\frac{1}{4}a^2-b)}$.

Therefore, if b be positive and x real, b cannot exceed $\frac{1}{4}a^2$. If β be the smaller of the two roots, its mean value, therefore, is